Midterm Introduction to Logic (AI and Ma)

Wednesday 13 December, 2017, 9 – 11 AM

☞ Only write your student number at the top of the exam, not your name, so that we can grade anonymously. Also put your student number at the top of any additional pages.

☞ Put the name of your tutorial group (AI 1, . . . , AI 6, Ma c, Ma d or Ma e) at the top of the exam and at the top of any additional pages.

☞ Leave the first ten lines of the first page blank (this is where the calculation of your grade will be written).

☞ Use a blue or black pen (so no pencils, no red pens).

☞ With the regular exercises, you can earn 90 points. By writing your student number and tutorial group on all pages, you earn a first 'free' 10 points. With the bonus exercise, you can earn an additional 10 points. The grade is:
(the number of points you earned with the regular and bonus exercises + the first 'free' 10) divided by 10, with a maximum grade of 10.

Good Luck!

1: Translation into propositional logic (10 points) Translate the following sentences into *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key. Represent as much logical structure as possible.

a. The human body does not have enough oxygen unless the heart pumps blood around and the lungs work.

b. If you are ambitious and you want to go for a 'cum laude' bachelor's degree, then you have to get at least an 8 on average and you cannot take re-sit exams.

2: Translation into first-order logic (10 points) Translate the following sentences to *first-order logic*. Do not forget to provide the translation key—one key for the whole exercise. Represent as much logical structure as possible.

a. Aron and Bas both love Chris if and only if Chris hates neither Aron nor Bas.

b. Chris or both Dunya and Bas love Aron only if Aron and Bas are siblings.

3: Formal proofs (30 points) Give formal proofs of the following inferences. Don't forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.

a.
$$\begin{array}{|l} A \to C \\ \neg D \to \neg B \\ \hline (A \lor B) \to (C \lor D) \end{array}$$

b.
$$\begin{array}{|l} \neg(a = a \land P(b)) \\ a = b \\ \hline \neg P(a) \end{array}$$

c.
$$\begin{array}{|l} \\ \hline P \leftrightarrow (\neg P \to P) \end{array}$$

4: Truth tables (15 points) Use *truth tables* to answer the next questions. Make the full truth tables, and do not forget to draw explicit conclusions from the truth tables in order to explain your answers.

Order the rows in the truth tables as follows:

P	Q	R	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a = b	Medium(a)	Large(b)	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a. Is $((Q \leftrightarrow \neg R) \vee (P \leftrightarrow \neg Q)) \vee (\neg P \leftrightarrow \neg R)$ a *tautology*?

b. Is the sentence Medium(a) $\leftrightarrow$ ¬Large(b) a *logical consequence* of the sentence a = b? Indicate the spurious rows in the truth table.

5: Normal forms of propositional logic (15 points)

a. Provide a negation normal form (NNF) of the sentence:
$\neg(P \leftrightarrow Q) \rightarrow \neg\neg(P \rightarrow R)$.

b. Provide a conjunctive normal form (CNF) of the sentence:
$\neg(P \vee R) \vee (Q \wedge (P \rightarrow S))$.

Indicate the intermediate steps. You do not have to provide justifications for the steps.

6: Set theory (10 points) Consider the following three sets: $A = \emptyset$, $B = \{\emptyset, \{\emptyset\}\}$ and $C = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$. For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

a. $A \in B$

b. $B \subset C$

c. $B \in C$

d. $\emptyset \subset A$

e. $\{\emptyset\} \in B \cap C$

f. $(A \cap B) \cap C \subseteq \emptyset$

g. $(A \cap B) \cap C = \{\emptyset\}$

h. $(A \cup B) \cup C = C$

i. $(B \setminus C) \subset A \cap B$

j. $C \setminus B = B \cup A$

7: Bonus question (10 points) Give a formal proof for $(P \rightarrow Q) \vee (Q \rightarrow R) \vee (R \rightarrow P)$. Don't forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.